

The breakup of a liquid in a turbulent stream of a dispersion medium is an inseparable part of all technological processes connected with the formation of aerosol and emulsion systems. For the case of isotropic turbulence, when the dynamics of the process of variation of the size distribution  $N(v, t)$  of particles of the disperse phase is determined only by the processes of their breakup, the function  $N(v, t)$  is a solution of the kinetic equation

$$\frac{\partial N(v, t)}{\partial t} = \int_v^{\infty} f(\omega) n(v, \omega) N(\omega, t) d\omega - f(v) N(v, t), \quad (0.1)$$

where  $f(v)$  is the frequency of breakup of particles in the size range  $(v, v + dv)$ ;  $n(v, \omega)$  is the probability of the formation of a particle in the size range  $(v, v + dv)$  in the breakup of a particle with a size in the range  $(\omega, \omega + d\omega)$ .

The purpose of the present article is to determine the functions  $f(v)$  and  $n(v, \omega)$ , solve Eq. (0.1), and analyze the results obtained. According to the author's information, the problem of investigating the dynamics of the process of breakup of a liquid has not been analyzed before in such a statement.

### 1. Determination of the Breakup Frequency

To find the function  $f(v)$  we use the following model. We shall assume that the breakup of a single particle in a turbulent stream is fully determined by the fluctuations of energy dissipation in its vicinity. Now if the value of the dissipation energy averaged over a volume on the order of the size of a particle exceeds the critical value  $a(v)$  then the act of breakup occurs. Henceforth we assume that the distribution of energy dissipation in the vicinity of a breaking-up drop is uniform with an average value  $\varepsilon(t)$ .

Since, according to the adopted model, the breakup frequency can be treated as the probability of the random process  $\varepsilon(t)$  crossing the level  $a(v)$  per unit time, under the condition that  $\varepsilon(t) < a(v)$  at the starting time, we write it in the form

$$f(v) = \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} \frac{P\{\varepsilon(t) < a(v), \varepsilon(t + \Delta t) > a(v)\}}{P\{\varepsilon(t) < a(v)\}}. \quad (1.1)$$

The numerator on the right side of this equation corresponds to the probability that the average value of the energy dissipation in the vicinity of the particle under consideration at the time  $t$  is less than, and at  $t + \Delta t$  is greater than, the quantity  $a(v)$ .

Expanding  $\varepsilon(t + \Delta t)$  in a series in  $\Delta t$  and taking the process as stationary, we convert the right side of Eq. (1.1) to the form

$$f(v) = \frac{\int_0^{\infty} \varepsilon p_2(a(v), \varepsilon) d\varepsilon}{\int_0^{\infty} p(\varepsilon) d\varepsilon}$$

where  $p_2(\varepsilon, \dot{\varepsilon})$  is the combined distribution density of the quantity  $\varepsilon(t)$  and its rate of change  $\dot{\varepsilon}(t)$  at the same time;  $p(\varepsilon)$  is the one-dimensional distribution of the quantity  $\varepsilon(t)$ . To determine the probability  $p_2(\varepsilon, \dot{\varepsilon})$  we use the well-known relation [1]

$$p_2(\varepsilon, \dot{\varepsilon}) = \lim_{\Delta t \rightarrow 0} \Delta t g_2\left(\varepsilon + \frac{\Delta t}{2} \dot{\varepsilon}, \varepsilon - \frac{\Delta t}{2} \dot{\varepsilon}\right) \quad (1.2)$$

where  $g_2(\varepsilon_1, \varepsilon_2)$  is the combined distribution density of the random quantity  $\varepsilon(t)$  at different times.

On the basis of an analysis of theoretical and experimental results on the investigation of the process of energy dissipation, it was concluded in [2] that  $p(\varepsilon)$  is well approximated by a logarithmic normal distribution law. Following this result, we write  $p(\varepsilon)$  in the form

$$p(\varepsilon) = \frac{1}{\sqrt{2\pi}\alpha\varepsilon} \exp\left\{-\frac{1}{2\alpha^2}(\ln \kappa\varepsilon)^2\right\}, \quad \alpha^2 = \ln\left(\frac{\sigma^2_\varepsilon}{\bar{\varepsilon}} + 1\right), \quad \kappa = \frac{1}{\bar{\varepsilon}} \exp\left\{\frac{\alpha^2}{2}\right\}, \quad (1.3)$$

where  $\bar{\varepsilon}$  and  $\sigma^2_\varepsilon$  are the mean value and the dispersion of the process of energy dissipation. Assuming that the process  $\varepsilon(t)$  is interconnected one-to-one with a stationary Gaussian process  $x(t)$  with a zero mean value, a dispersion  $\alpha^2$ , and a correlation function  $R^2_x$  through the transformation  $x = \ln(\kappa\varepsilon)$  of the two-dimensional distribution density  $g_1(x(t), x(t + \tau))$ , we obtain

$$g_2(\varepsilon_1, \varepsilon_2) = \frac{1}{2\pi\alpha^2\beta\varepsilon_1\varepsilon_2} \exp\left\{-\frac{1}{2\alpha^2\beta^2}[(\ln \kappa\varepsilon_1)^2 - 2R_x \ln \kappa\varepsilon_1 \cdot \ln \kappa\varepsilon_2 + (\ln \kappa\varepsilon_2)^2]\right\}, \quad \varepsilon_1 = \varepsilon(t), \quad \varepsilon_2 = \varepsilon(t + \tau), \quad \beta^2 = 1 - R_x^2. \quad (1.4)$$

In such a transformation the correlation functions of the processes  $\varepsilon(t)$  and  $x(t)$  satisfy the equality

$$R_\varepsilon^2(\tau) = \frac{\exp\{\alpha^2 R_x^2(\tau)\} - 1}{\exp\{\alpha^2\} - 1}. \quad (1.5)$$

To calculate the combined distribution density  $p_2(\varepsilon, \dot{\varepsilon})$  one has to know the behavior of the correlation function  $R^2_\varepsilon(\tau)$  only at small values of  $\tau$ . Since the process of energy dissipation is a physically differentiable process with a correlation function differentiable at zero, we shall assume that at small values of  $\tau$

$$R_\varepsilon^2(\tau) \approx 1 - \left(\frac{\tau}{T_0}\right)^2 \quad (1.6)$$

[ $T_0$  is the time constant of correlation for the process  $\varepsilon(t)$ ]. Under the assumption that the correlation time depends only on the mean value of the energy dissipation and the viscosity  $\nu$  of the dispersion medium, from dimensionality theory we obtain  $T_0 \approx \sqrt{\nu/\bar{\varepsilon}}$ .

Relations (1.4)-(1.6) enable us to expand the right side of Eq. (1.2). We write the result of the calculations in the form

$$p_2(\varepsilon, \dot{\varepsilon}) = \frac{T_0}{2\pi\alpha c \varepsilon^2} \exp\left\{-\frac{T_0^2}{2c^2} \left(\frac{\dot{\varepsilon}}{\varepsilon}\right)^2 - \frac{1}{2\alpha^2}(\ln \kappa\varepsilon)^2\right\}, \quad c = 1 - \exp\{-\alpha^2\}. \quad (1.7)$$

Substituting (1.7) and (1.3) into (1.1) and performing the necessary transformations, we obtain the expressions for the breakup frequency:

$$f(y) = \frac{c}{\sqrt{2\pi} T_0 \alpha} \frac{d}{dy} \ln \Phi(y), \quad (1.8)$$

$$\Phi(y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^y e^{-\frac{1}{2}y^2} dy, \quad y = \frac{1}{\alpha} \ln(\kappa a(v)).$$

To find the quantity  $a(v)$  we use a model of the mechanism of breakup of drops based on a comparison of capillary forces and the forces of viscous friction. Such a model has been discussed repeatedly [3] in connection with the determination of the minimum size of the radius  $r_0$  of drops breaking up in a turbulent stream,

$$r_0 = \gamma \frac{\sigma}{(\rho \sqrt{\varepsilon v})}, \quad (1.9)$$

where  $\sigma$  is the interphase surface tension;  $\nu$  and  $\rho$  are the kinematic viscosity and the density of the dispersion phase;  $\gamma$  is a numerical factor on the order of unity.

Replacing  $\bar{\varepsilon}$  by  $a(v)$  in (1.9) and converting from the radii to the volumes of the drops breaking up, we obtain

$$a(v) = \left[ \gamma \left( \frac{4\pi}{3} \right)^{1/3} \frac{\sigma}{\rho} \right]^2 \frac{1}{v v_0^{2/3}}, \quad v_0 = \frac{4}{3} \pi r_0^3. \quad (1.10)$$

We determine the quantity  $\alpha$  appearing in (0.1) on the basis of the result of [4], where the equality

$$\sigma_g^2 = 0.4 \bar{\epsilon}^3$$

is obtained from the assumption that Millionshchikov's hypothesis is valid for the velocity pulsations in a field of isotropic turbulence. Substituting (1.10) into (1.8) and calculating the quantity  $\alpha$ , we reduce the expression for the breakup frequency to the form

$$f(x) = \frac{1}{2.03 \sqrt{2\pi} T_0} \frac{d}{dx} \ln \Phi(x), \quad x = -1.1 \ln \left( 1.3 \frac{v}{v_0} \right). \quad (1.11)$$

This function, positive definite on the real axis, decreases monotonically with an increase in  $x$ . It can be approximated with 5% accuracy by expressions of the type

$$2.03 \sqrt{2\pi} T_0 f(x) = \begin{cases} -x + \frac{0.798}{1 - 0.65x} & \text{for } x \leq 0, \\ \frac{1}{\sqrt{2\pi}} \exp \left\{ -\frac{x^2}{2} \right\} (1 + \exp \{-1.65x\}) & \text{for } x \geq 0. \end{cases} \quad (1.12)$$

The concept of the breakup frequency enables us to arrive at a definition, from the probabilistic standpoint, of the minimum size of drops which can break up in a turbulent stream. It seems natural to connect the minimum linear size  $r_1$  of breaking-up drops with the critical breakup frequency  $f_1$ , taking the latter as equal to a certain small number ( $0 < f_1 \leq 1$ ). Identifying  $f(x)$  with  $f_1$  and  $v$  with  $v_1$  in (1.12), and considering that  $f(x)$  assumes small values for  $x > 1$ , we obtain

$$\begin{aligned} r_0/r_1 &\approx 0.92 \exp \{0.41 \sqrt{-\ln z}\}, \\ z &= 2.03 \sqrt{2\pi} T_0 f_1. \end{aligned}$$

The value of the minimum size  $r_0$  for breaking-up drops generally accepted today exceeds severalfold the quantity  $r_1$  ( $f_1 \leq 0.01$ ). Here the ratio  $r_0/r_1$  increases monotonically with an increase in the intensity of turbulization of the stream (Fig. 1, curve 1). Equality of  $r_0$  and  $r_1$  is reached when the condition  $T_0 f_1 = 0.12$  is satisfied, which corresponds to values of  $f_1 > 1$  in the region of developed turbulence. Curves 2 and 3 correspond to the ratios  $r^*/r_1$  of two values of the drop sizes calculated at different breakup frequencies ( $0.01 \geq f_1 < f_1^*$ ;  $f_1/f_2 = 10^{-3}$  and  $10^{-4}$ ) as functions of the intensity of turbulization of the stream. As seen from these graphs, a decrease in the value of  $f_1$  by even three orders of magnitude leads to a change in  $r_1$  by a factor of no more than 1.7. Such a weak dependence of  $r_1$  on  $f_1$  enables us (without the danger of significantly affecting the value of  $r_1$ ) to choose any of the values of  $10^{-2}$ ,  $10^{-3}$ , or even  $10^{-4}$  for  $f_1$ , satisfying our concepts of the smallness of the frequency of breakup of drops, the breakup of which is hardly observed.

## 2. Determination of the Function $n(v, \omega)$

In small regions (on the order of the drop size) of a turbulent stream a large number of hydrodynamic situations can occur randomly, corresponding to different mechanisms of breakup, and hence to different variants of breakup. Therefore, the function  $n(v, \omega)$  describing the breakup of a single drop is probabilistic in its nature, which is taken into account in its determination.

Suppose that a single drop of volume  $\omega$  can give rise to up to  $k$  daughter drops with volumes  $v_i$  ( $i = \overline{1, k}$ ). We shall characterize the probability of formation of these drops by a multidimensional distribution density  $p(v_1, v_2, \dots, v_k, \omega)$ . In this case the probability of formation of a drop of a fixed volume  $v$  will be

$$\begin{aligned} n(v, \omega) &= \sum_{i=1}^k \int_0^\omega \dots \int p(v_1, \dots, v_{i-1}, v_i, v_{i+1}, \dots, v_k, \omega) dv_1 \dots \\ &\quad \dots dv_{i-1} dv_{i+1} \dots dv_k. \end{aligned} \quad (2.1)$$

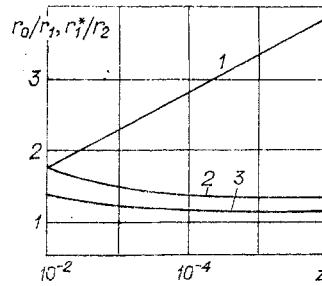


Fig. 1

By virtue of the equal justification of all  $v_i$ , the probability density  $p(v_1, \dots, v_k, \omega)$  must be a symmetric function of these variables, while all the integrals in (2.1) are equal to each other. Since the value of each integral in (2.1) is a one-dimensional distribution density  $g(v, \omega)$ , we write Eq. (2.1) in the form  $n(v, \omega) = kg(v, \omega)$ .

Numerous observations of acts of the breakup of single drops [3] under different hydrodynamic conditions indicate that several (most often two) approximately equal drops and one or several smaller drops, so-called satellites, are usually formed in a breakup. But in none of the papers has a correlation been noted between the different variants of breakup and the sizes of the broken-up drops. The absence of such a connection allows us to assume that the function  $n(v, \omega)$ , for fixed properties of the disperse and dispersion phases, depends only on the ratio of the quantities  $v$  and  $\omega$ , and it can be written in the form

$$n(v, \omega) = k \frac{1}{\omega} g\left(\frac{v}{\omega}\right).$$

To conserve the total volume of the daughter drops, the function  $n(v, \omega)$  must satisfy the condition

$$\int_0^{\omega} vn(v, \omega) dv = \omega_s \quad (2.2)$$

from which it follows that the mean value of the distribution density  $g(y)$  is

$$\bar{y} = \int_0^1 yg(y) dy = \frac{1}{k}.$$

On the basis of the above-indicated observations of the breakups of single drops one can assume that the function  $g(y)$  is, as a rule, a bimodal function with two clearly expressed maxima, one of which lies in the region of the sizes of the satellites and the other in the region of the sizes of the larger drops formed in the breakup. Taking the function  $g(y)$  as bimodal, we represent it as a sum of two weighted unimodal distribution densities  $g_1(y)$  and  $g_2(y)$ , defined in the interval  $(0, 1)$  and having mean values  $y_1 = v_1/\omega$  and  $y_2 = v_2/\omega$ :

$$n(v, \omega) = k_1 \frac{1}{\omega} g_1\left(\frac{v}{\omega}\right) + k_2 \frac{1}{\omega} g_2\left(\frac{v}{\omega}\right), \quad k = k_1 + k_2. \quad (2.3)$$

In their physical meaning the quantities  $k_1$  and  $k_2$  correspond to the mathematical expectations of the total numbers of daughter drops formed in the vicinities of the sizes  $y_1$  and  $y_2$ . To satisfy the condition (2.2), the values of  $k_i$  and  $y_i$  ( $i = 1, 2$ ) must satisfy the equality  $k_1 y_1 + k_2 y_2 = 1$ . In the limiting case, when the dispersions of the distribution densities  $g_1(y)$  and  $g_2(y)$  approach zero, which corresponds to the absence of scatter in the sizes of the daughter drops (the determinate model of breakup), Eq. (2.3) can be reduced to the form

$$n(v, \omega) = k_1 \delta(v - y_1 \omega) + k_2 \delta(v - y_2 \omega), \quad (2.4)$$

where  $\delta(x)$  is the delta function.

If we take the function  $n(v, \omega)$  not as bimodal but as multimodal or, in the case of its bimodality, we do not "compress" each of its component distributions to the mean value but approximate it by a suitable discrete distribution, then, instead of (2.4), we obtain

$$n(v, \omega) = \sum_{i=1}^n k_i \delta(v - y_i \omega), \quad \sum_{i=1}^n k_i = k, \quad \sum_{i=1}^n k_i y_i = 1. \quad (2.5)$$

### 3. Solution of the Kinetic Equation

We analyze the solution of Eq. (0.1) only for the case when a drop is broken exactly into halves without the formation of satellites. The function  $n(v, \omega)$  corresponding to this breakup model, in accordance with its definition (2.4), has the form

$$n(v, \omega) = 2\delta(v - 0.5\omega). \quad (3.1)$$

Substituting (3.1) into (0.1) and calculating the integral, we obtain

$$\partial N(v, t) / \partial t = 2f(2v)N(2v, t) - j(v)N(v, t). \quad (3.2)$$

From the structure of this equation it is seen that its general solution can be represented in the form of a sum of independent particular solutions with discrete spectra. In this case, if  $v$  is the maximum size of the drop in the initial condition for any of these solutions, then the solution will be determined at the points  $2^{-j}v$  ( $j = 0, 1, \dots$ ). Naturally, the initial condition for this solution must also be determined at these points.

Let us examine the procedure for obtaining one of these solutions. Writing Eq. (3.2) for each of the points of determination of the solution, we have the infinite system of equations

$$\begin{aligned} \frac{\partial}{\partial \tau} N_1 &= -\varphi_1 N_1, \quad \frac{\partial}{\partial \tau} N_i = 2\varphi_{i-1} N_{i-1} - \varphi_i N_i, \quad i = 2, 3, \dots, \\ \tau &= \frac{t}{2.03 \sqrt{2\pi} T_0} N_i = N(z_i, \tau), \quad z_i = 2^{-i} \frac{v}{\omega_0}, \quad \varphi_i = 2.03 \sqrt{2\pi} T_0 f(z_i), \end{aligned}$$

and solving them successively, starting with the first, we obtain

$$\begin{aligned} N_1(\tau) &= c_1 e^{-\varphi_1 \tau}, \quad N_i(\tau) = \sum_{j=1}^{i-1} c_j \alpha_{ij} e^{-\varphi_j \tau} + c_i e^{-\varphi_i \tau}, \quad i = 2, 3, \dots, \\ \alpha_{ij} &= 2^{i-j} \left( \prod_{k=j}^{i-1} \varphi_k \right) \left( \prod_{k=j+1}^i (\varphi_k - \varphi_j) \right)^{-1}. \end{aligned} \quad (3.3)$$

The constant coefficients  $c_i$  are determined by the initial conditions and are the solution of the system of linear equations

$$N_1(0) = c_1, \quad N_i(0) = \sum_{j=1}^{i-1} c_j \alpha_{ij} + c_i, \quad i = 2, 3, \dots,$$

by the successive solution of which we find

$$c_1 = N_1(0), \quad c_i = N_i(0) - \sum_{j=1}^{i-1} c_j \alpha_{ij}, \quad i = 2, 3, \dots$$

On the basis of the particular solution (3.3) we can construct an algorithm for determining the general solution of Eq. (3.2). First we consider the case when the initial condition for Eq. (3.2) is given in the form of a lattice function  $F(v)$ , determined at an ordered system of points  $v_i$  satisfying the condition  $v_i > v_{i+1}$ . This function can always be represented in the form of a sum of lattice functions, each of which will determine the initial condition for a particular solution (3.3). We calculate the components of the sum using the following recurrent procedure. In the first step we isolate the initial condition  $F_1(v)$  for the first particular solution, which is determined at the points  $2^{-i}v_1$  ( $i = 0, 1, 2, \dots$ ). Then we calculate the difference  $F(v) - F_1(v)$  and, identifying it with the original function  $F(v)$ , we similarly find the initial condition for the second particular solution, etc. The sum of the particular solutions with these initial conditions corresponds to the solution of Eq. (3.2) with the initial condition in the form of the function  $F(v)$ .

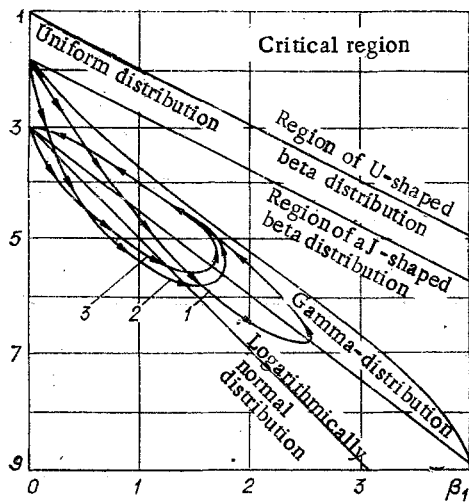


Fig. 2

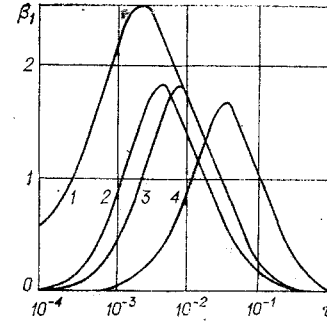


Fig. 3

If the initial condition is given in the form of a continuous function, then by approximating it with a lattice function we reduce the problem to the preceding one.

In investigation the solution of Eq. (2.5) it is of interest to examine its dependence both on the breakup time and on the form of the initial conditions. For this purpose we calculated solutions of Eq. (3.2) with initial conditions in the form of a monodisperse distribution concentrated at the point  $z = v/\omega_0$  and of a uniform distribution in the interval  $[z, z/2)$ . The value of  $z$  was varied in the range of  $10^{-10^6}$ . In the calculations the uniform initial distribution was approximated by a lattice function with 10 ordinates of equal amplitude, distributed with a uniform step on a logarithmic scale of  $z$ .

It was established by a direct test that a twofold increase in the degree of quantization of the initial condition leads to no more change in the results presented below than in the second decimal place.

On the basis of the results obtained, we calculated the dependence of the first four moments for the distribution density corresponding to the solution of Eq. (3.2) on the breakup time and calculated the coordinates of the Johnson-Pearson diagram [5],

$$\beta_1(\tau) = \left( \frac{\mu_3(\tau)}{\mu_2^{1.5}} \right)^2, \quad \beta_2 = \left( \frac{\mu_4}{\mu_2^2} \right)^2,$$

where  $\mu_i$  is the centered  $i$ -th moment of the distribution density.

In the calculation of the above-indicated quantities the discrete solutions of Eq. (3.2) were approximated through linear interpolation of their neighboring values. All the calculations were made on a logarithmic scale of the variable  $z$ .

Some of the characteristic hodographs for the size distribution densities of particles, corresponding to solutions of Eq. (3.2) with different initial conditions, are presented in Fig. 2 (1 is a monodisperse distribution at the point  $z = 10^6$ , while 2 and 3 are uniform distributions of  $z$  in the intervals  $(0.5 \cdot 10^6 - 10^6)$  and  $(0.5 \cdot 10^3 - 10^3)$ ). All the hodographs converge asymptotically to a normal distribution law on the logarithmic scale of drop sizes. This asymptotic behavior is retained for all the calculated variants of solutions.

The result obtained is generalized to an arbitrary initial condition if one considers that the latter can always be approximated with any degree of accuracy by a sum of monodisperse distributions. Since each of the particular solutions corresponding to a monodisperse initial condition converges to a normal distribution, on the basis of the central-limit theorem of probability theory, the sum of particular solutions will also converge to a normal distribution on a logarithmic scale or to a logarithmically normal distribution on a linear scale of particle sizes.

This result fully coincides with the conclusion of [6], where an asymptotic solution of Eq. (0.1) was analyzed under the condition that  $n(v, \omega)$  is given in the form (3.1), while the function  $f(v)$  is constant and does not depend on the particle sizes; the desire to verify

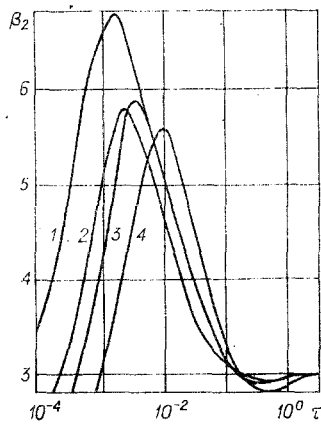


Fig. 4

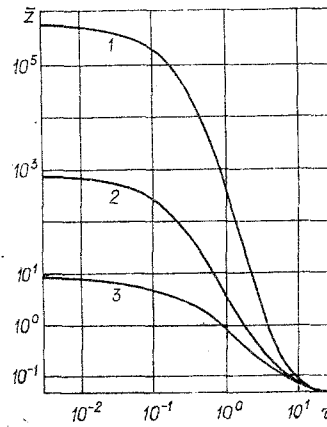


Fig. 5

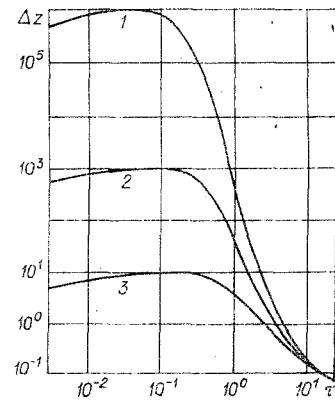


Fig. 6

this conclusion for the case when  $f(v)$  has a power-law dependence on the size of the particles breaking up is also expressed there. The results obtained allow us to state that the results of [6] are valid not only when the function  $f(v)$  has a power-law character, but even for the stronger dependence defined by Eq. (1.11).

The rate and the time of emergence of the solution at its asymptotic value are seen well in Figs. 3 and 4, where the time dependence of the parameters  $\beta_1$  and  $\beta_2$  is shown for solutions with different initial conditions: 1) a monodisperse distribution at the point  $z = 10^6$ ; 2-4) uniform distributions of  $z$  in the interval  $(0.5-1] \cdot 10^6$ ,  $(0.5-1] \cdot 10^3$ , and  $(5-10]$ . As seen from Figs. 3 and 4, at  $\tau > 1$  the solution can be considered as practically logarithmically normal ( $\beta_1 = 0$ ,  $\beta_2 = 3$ ).

In Fig. 5 we present graphs of the average drop size as a function of the breakup time, calculated for different initial conditions: 1-3) initial distributions of  $z$  in the intervals  $(0.5-1] \cdot 10^6$ ,  $(0.5-1] \cdot 10^3$ , and  $(5-10]$ . It is seen that even for greatly differing initial conditions the mean values of the solutions at  $\tau > 10$  become practically the same.

To analyze the dependence of the width of the spectrum of drop sizes on the breakup time and the initial conditions, we calculated the graphs presented in Fig. 6. The width  $\Delta z$  of the size region into which 99% of the total number of drops fall is laid out along the ordinate axis. Curves 1-3 correspond to uniform distributions of  $z$  in the intervals of  $(0.5-1] \cdot 10^6$ ,  $(0.5-1] \cdot 10^3$ , and  $(5-10]$  in the original emulsion. It is seen that, just as for the time dependence of the mean values of the size distribution of the particles, the values of  $\Delta z$  become practically the same at  $\tau \geq 10$ . In this case the ratio  $\Delta z/\bar{z}$  has a value of about 0.5 in the region of  $\tau \geq 10$ .

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